Realistic and effective interactions in the study of nuclear matter [1]

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Abstract

The equation of state of symmetric nuclear matter is addressed starting both from a realistic interaction derived from nucleon-nucleon scattering processes and from a low-momentum effective potential. The approach is based on finite temperature Green's functions. The internal energy per particle is estimated from the summation of diagrams and through the Galitskii-Koltun's sum rule.

At present, there is not a unique and reliable theory of nuclear interactions. From the study of nucleon-nucleon scattering processes different models for the basic interaction between nucleons have been developed, but difficulties arise when they are used in calculations which aim to reproduce the properties of bound nuclei. These models all reproduce the experimental N-N phase shifts up to the threshold for pion production with high accuracy, but differ in the short-wavelength region, where less experimental constraints are available.

In deriving the properties of dense nuclear matter, if we start from a bare N-N potential we have to take into account the complex correlations induced by strong interactions. In parallel with the development of diagram summation techniques, a variety of effective potentials have been proposed. Apart from the traditional phenomenological N-N forces, to be used in a mean-field approximation, such as the Skyrme and the Gogny interactions, in the last years renormalized low-momentum potentials have been introduced [1]. They are restricted to a subspace of the Hilbert space, the so-called model space, in which they incorporate the short-range correlations between nucleons. If the cutoff is sufficiently low, they turn to be independent of the starting N-N interaction.

In this work we present a self-consistent scheme based on finite temperature Green's functions and apply it to study the equation of state of symmetric nuclear matter. We first consider a bare nucleon-nucleon interaction, namely the CD-Bonn potential, and compare the results with the case in which a renormalized potential V_{low-k} [2] is used.

It is possible to construct a consistent approximation starting from a suitably chosen generating functional [3]. Such approximation schemes automatically fulfill thermodynamic relations, including the Hugenholz-Van Hove and Luttinger identities. For nuclear interactions the generating functional must at least include ladder-type diagrams: this choice leads to the in-medium T-matrix, the approximation scheme adopted in our study.

The in-medium two-particle scattering matrix T is defined as (for simplicity we skip spin and isospin indices, as well as energy and momentum dependence):

$$T = V + V\, G_2^{nc}\, T \ . \tag{1}$$

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Here V is the interaction potential and G_2^{nc} is the *non correlated* two-particle Green's function, just constructed as a product of two dressed single-particle propagators.

The T-matrix accounts for the multiple scattering between nucleons and thus permits to resum the short range correlations induced in the dense system. We remark that the use of the dressed propagator implies the presence of a nontrivial dispersive self-energy which leads to a broad spectral function [5]. The full two-particle propagator is then approximated as follows

$$\mathcal{G}_2 = G_2^{nc} + G_2^{nc}\, T\, G_2^{nc} + \ exchange\, terms \ . \tag{2}$$

Eqs. (1) and (2), together with the Dyson equation, form a scheme in which all ingredients are calculated iteratively: the scattering matrix, the single-particle self-energy, which is expressed in terms of the scattering matrix, and the single-particle propagator.

Most of the properties of the system can be derived from these quantities. The (total) internal energy per particle can be calculated as the expectation value of the Hamiltonian $H = H_{kin} + H_{pot}$. The two terms read (we now make explicit the momentum and energy dependences)

$$\langle H_{kin} \rangle = \mathcal{V} \int \frac{d^3p}{(2\pi)^3} \frac{d\omega}{2\pi} \frac{\mathbf{p}^2}{2m} A(\mathbf{p},\omega) f(\omega) \,, \tag{3}$$

where $\mathcal{V}$ represents the volume, $A(\mathbf{p},\omega)$ is the spectral function and $f(\omega)$ the Fermi-Dirac distribution, and

$$\begin{aligned} \langle H_{pot} \rangle = \frac{\mathcal{V}}{2} \int \frac{d^3P}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{d\Omega}{2\pi}\, b(\Omega) \\ \times \mathrm{Im} \left\{ \left(\langle \mathbf{k}|T^R(\mathbf{P},\Omega)|\mathbf{k}\rangle - \langle \mathbf{k}|T^R(\mathbf{P},\Omega)|-\mathbf{k}\rangle \right) G_2^{nc\ R}(\mathbf{P},\mathbf{k},\Omega) \right\} \ , \end{aligned} \tag{4}$$

where Ω and $\mathbf{P}$ are the total energy and momentum of the pair of nucleons, $\mathbf{k}$ the exchanged momenta and $b(\Omega)$ the Bose-Einstein distribution.

An alternative and simpler way to determine the energy is the Galitskii-Koltun's sum rule [6, 7]

$$\frac{E}{N} = \frac{1}{\rho} \int \frac{d^3p}{(2\pi)^3} \frac{d\omega}{2\pi} \left[\frac{\mathbf{p}^2}{2m} + \omega \right] A(\mathbf{p},\omega) f(\omega) \ , \tag{5}$$

which for conserving approximations is equivalent to the direct calculation (3)+(4). In the presence of three-body forces, however, this is not valid and the expectation value of the Hamiltonian has to be computed to estimate the internal energy of the correlated system.

The ladder expansion can be as well derived from a generating functional $\Phi[G,V]$, from which pressure and entropy at finite temperature can be obtained [4].

We performed calculations for the internal energy per particle in two ways: from the Galitskii-Koltun's sum rule (5) and from diagram summation, i.e. eqs. (3) and (4). For the two methods we employ both a realistic nucleon-nucleon interaction, the CD-Bonn potential, and the effective interaction V_{low-k} from

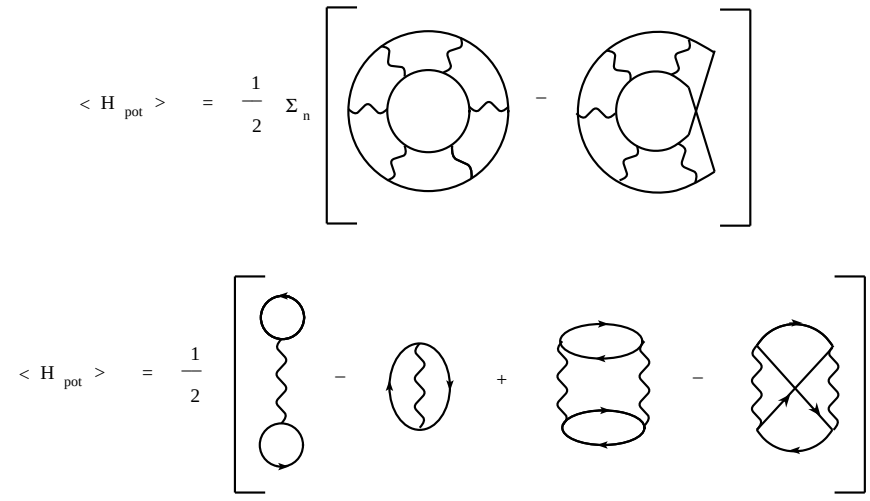

Figure 1: Diagrammatic expression for the expectation value of the potential energy in the case of the full T-matrix approach (upper figure, the sum runs over the number of interaction lines n in the diagram) and for a second order approximation (lower figure).

[2]. In the first case we consider the whole expansion of the propagator in ladder-type diagrams, which lead to the diagrammatic expression for the interaction energy that appears in Fig. 1 (up). The energy depends on dressed propagators (solid lines in the figure) and on the two-body potential (wavy lines). In the case of the effective interaction we only include diagrams up to the second order (Born approximation), which are shown in Fig. 1 (down). Since V_{low-k} already accounts for the correlations in the dense system the higher order diagrams do not yield significant contributions and can be neglected [2].

We restrict ourselves to symmetric nuclear matter at zero temperature, though we remark that this formalism can be applied to finite temperature systems as well. Results for the internal energy per particle as a function of density are shown in Fig. 2, in which calculations from the diagram summation are compared to the Galitskii-Koltun's sum rule. The calculations for the two potentials are in agreement up to $0.8\rho_0$. At higher densities, the low-momentum calculations give too attractive energies and differ substantially from the full T-matrix ones, exhibiting no minimum. On the contrary, the full calculation with a realistic nucleon-nucleon potential shows a saturation point, which however does not coincide with the experimental value (as expected since three-body forces are not included.)

Concluding, we have studied the equation of state of symmetric nuclear matter both with the two-body CD-Bonn potential and with a renormalized interaction V_{low-k}. When the effective potential is used, the energy behavior

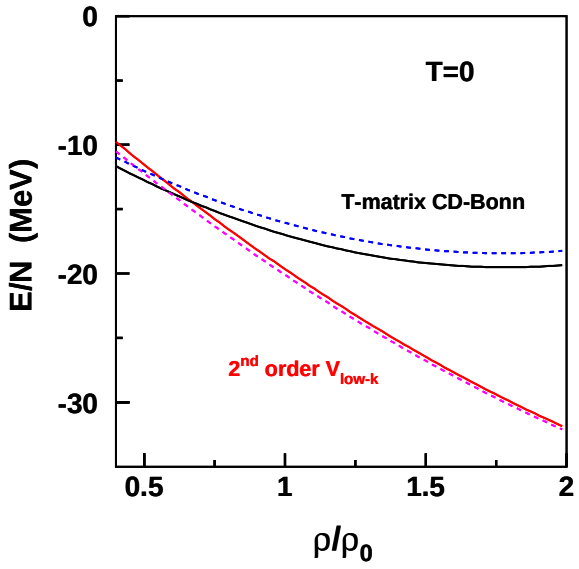

Figure 2: The internal energy per particle at zero temperature as a function of the density (in units of the empirical saturation density $\rho_0 = 0.16$ fm^{-3}). The solid lines represent the expectation value of the Hamiltonian (3)+(4), the dashed lines are the results obtained from the sum rule (5).

shows that V_{low-k} alone at high densities does not correctly account for all correlations. This would not happen if a larger cutoff were chosen, even if in this case V_{low-k} would depend on the starting nucleon-nucleon realistic force. This dependence, which can be amplified or suppressed by changing the cutoff, can possibly help in testing the reliability of the different models of the bare nucleon-nucleon interaction.

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